A new method for calculating light's bending angle in gravitational fields

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Abstract

Schwarzschild's solution has been used to calculate light's bending angle in gravitational fields by performing an elliptic integration. However, because Schwarzschild's solution includes non-linear distortion of radial axis, it is very difficult to calculate light's bending angle. Then, a new method was discovered to calculate light's bending angle. The new method is based on the assumption that light advances changing light's speed and bending light's advancing direction in gravitational fields. The validity of the new method is proved by showing that non-linear distortion of radial axis in Schwarzschild's solution corresponds to light's bending angle derived based on the assumption of the new method. Consequently, the new method is identical to Schwarzschild's solution, but simple, easy and exact to calculate light's bending angle.

1. Introduction

The new method is based on the assumption that radial axis doesn't be distorted, but light advances changing light's speed and advancing direction in gravitational fields. In other words, non-linear distortion of radial axis in Schwarzschild's solution is replaced to light's bending angle which is calculated using the new method.

2. Relation between Schwarzschild's solution and the new method

R' : Distance from the center of gravitational fields

M : Mass in the center of gravitational fields

C : Light's speed in static system

C′ : Light's speed in gravitational fields

θ : Light's rotation angle around the center

ω : Rotation angle of light's advancing direction

d : Extremely infinitesimal change i.e. differential

G : Gravitational constant

$\alpha = 2GM/C^2$

Schwarzschild's solution is as the following.

(1) $-dS^2 = C^2(1- \alpha/R')dt'^2 - dR'^2/(1 - \alpha/R') - R'^2(d\theta^2 + \sin^2\theta d\Phi^2)$

$\alpha = 2GM/C^2$, $C'^2 = C^2(1- \alpha/R')$, $d\Phi = 0$

When the parameters are set as the above, the equation (1) is rewritten as the following.

(2) $-dS^2 + (\alpha/R')dR'^2/(1-\alpha/R') = C'^2dt'^2 - dR'^2 - R'^2d\theta^2$

The right side of the above equation (2) is equal to 0 as shown in Fig.1.

Here, the following equation (3) is presented.

(3) $(d\omega/d\theta) = (\alpha/R')/(1-\alpha/R')$

In the section **4**, this equation (3) is derived based on the assumption of the new method.

Then, the equation (2) is rewritten as the following.

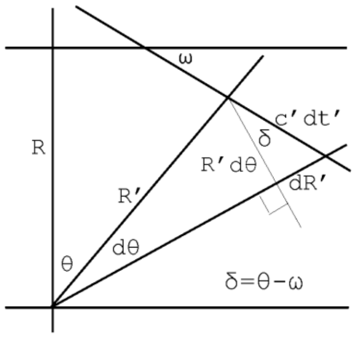

Fig.1

(4) $-dS^2 + (d\omega/d\theta)dR'^2 = (1 -/R')dt'^2 - dR'^2 - R'^2d\theta^2 = 0$

The above equation (4) proves that $(d\omega/d\theta)$ corresponds to the nonlinear distortion of the radial axis R' in dS^2. Because the right side of the equation (4) include no distortion of the radial axis R'.

3. How light's speed is changed in gravitational fields

It is assumed that an arbitrary location in gravitational fields belongs to an inertia system which has time axis t' with light's speed C and radial axis R' with relative speed $U(R')$ for static system. The static system has time axis t with light's speed C and radial axis R. Then, since the world space distance dS^2 is preserved for coordinate transformation to the inertia system from the static system, the following formula is formed.

C': Light's speed at the arbitrary position in the inertia system (gravitational fields)

C: Light's speed in the static system (non-gravitational fields)

i: Imaginary ($i^2 = -1$)

$$dS^2 = (iCdt)^2 = dS'^2 = (iCdt')^2 + (U(R)dt')^2$$

$$d\theta = 0 , \quad d\Phi = 0$$

Note that the inertia system to which the arbitrary location belongs is different by the location. And also the static system is non-gravitational fields.

Since light's advancing distance is invariant without depending on the inertia system, the following equation is formed.

$$C'dt' = Cdt.$$

Then, the following equation (6) is formed.

(6) $$C'^2 = C^2 - U^2 , \quad U = U(R)$$

On the other hand, when mass $m(\ll M)$ with speed v invades into gravitational fields of mass M from infinite remote location, the following equations are formed.

$g = GM/R^2$: Gravitational acceleration

v : Mass's speed at infinite remote location

V' : Mass's speed in gravitational fields

F : Gravitational force acting on mass m

$(iCdt')^2 + (Udt')^2 + (vdt')^2 = (dS')^2 = (iCdt')^2 + (V'dt')^2$

$(Udt')^2 + (vdt')^2 = (V'dt')^2$

$V'^2 = v^2 + U^2$

$mV'^2/2 = mv^2/2 + mU^2/2$

$d(mv^2/2)/dR = 0$

$d(mV'^2/2)/dR = d(mU^2/2)/dR = F = mg = mGM/R^2$

$dU^2/dR = 2GM/R^2$

$U^2 = 2GM/R$

Then, the equation (6) is rewritten as the following.

(7) $C'^2 = C^2 - 2GM/R$

When $C'^2 = 0$, $C^2 - 2GM/Rs = 0$, $Rs = 2GM/C^2$

Rs : Schwarzschild's radius

4. How light's advancing direction is changed in gravitational fields?

In Fig.2, light vertically crosses the location of the distance R from gravitational center of mass M. And then, light passes the location of the angle θ and the distance R' from gravitational center rotating light's advancing direction by the angle ω.

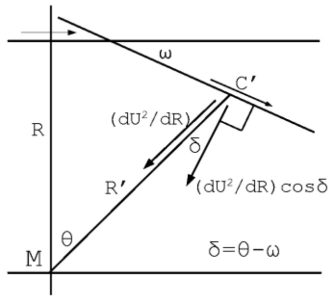

Fig.2

It is assumed that the rotating angular speed $d\omega/dt'$ of light's advancing direction in gravitational fields varies according to the following equation (8).

$\delta = \theta - \omega$

(8) $C'(d\omega/dt') = (dU^2/dR)\cos\delta$

$= C'(d\omega/d\theta)(d\theta/dt')$

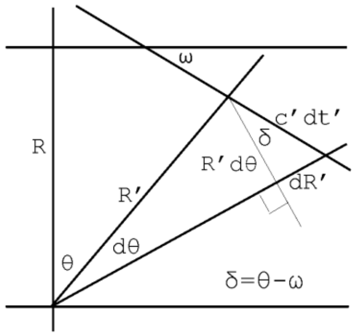

Fig.3

In Fig.3, the following equations are formed.

$d\theta/dt' = C'\cos\delta/R'$

Then, the equation (8) is rewritten as the following.

$d\omega/dt' = (d\omega/d\theta)(C'\cos\delta/R') = 2GM\cos\delta/C'R'^2$

$d\omega/d\theta = 2GM/C'^2R'$

When C'^2 is substituted by the equation (7), the equation (3) in section **2** is formed as the following equation (9).

(9) $d\omega/d\theta = (2GM/C^2R')/(1 - 2GM/C^2R') = (\alpha/R')/(1 - \alpha/R')$

$\alpha = 2GM/C^2$

5. Specific examples of light's bending angle

5.1 In the case of $Rs \ll R = R'\cos\theta$, $\alpha/R' \ll 1$ and $\omega \ll \theta$ as shown in Fig.4, the following equations are formed.

$d\omega/d\theta = (\alpha/R)\cos\theta$

$\omega_p = \int d\omega = \int_{-\pi/2}^{\pi/2} \int (\alpha/R)\cos\theta\, d\theta$

$= (\alpha/R)(\sin(\pi/2) - \sin(-\pi/2))$

$= 2\alpha/R' = 4GM/C^2R$

Light's bending angle ω_p passing the periphery of the sun may be calculated using the above equation.

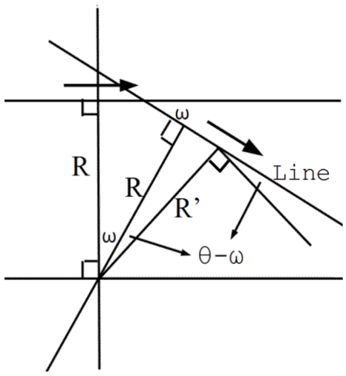

Fig.4

5.2 In the case of $R = \gamma Rs$, $2 < \gamma$, $\delta = \theta - \omega > 0$ and $d\delta/d\theta > 0$ in Fig.5, the following equations are formed.

$dR' = C'dt'\sin\delta$

$d\theta/dt' = C'\cos\delta/R'$

$dR'/dt' = (dR'/d\theta)(d\theta/dt') = C'\sin\delta$

$(dR'/d\theta)(C'\cos\delta/R') = C'\sin\delta$

$d(\log R')/d\theta = \sin\delta/\cos\delta = -d(\log(\cos\delta))/d\delta$

$\log R' = -\log(\cos\delta) + \log\ A$,

A: Integration constant

$R'\cos\delta = R'\cos(\theta - \omega) = A$, $\theta = \omega$

$R' = R = A$

(10) $R'\cos\delta = R$

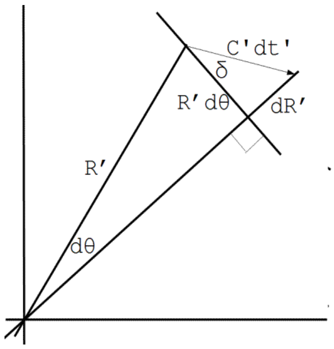

Fig.5

Then, the equation (9) is rewritten as the following.

(11) $d\omega/d\theta = (\alpha\cos\delta/R)/(1 - \alpha\cos\delta/R)$

$\alpha = 2GM/C^2$

According to the above equation (6) and Fig.3, the orbit

of light seems to be drawn using rulers.
And the equation (11) is rewritten as the following.

(12) $d\delta/d\theta = (\gamma - 2\cos\delta)/(\gamma - \cos\delta) > 0$

This seems to be a parabolic like orbit.
Approximately along the thick line in Fig.5, the angle δ increases as θ increases. The fold point of the thick line moves upward or downward on the line of $\theta = \pi/2$ as γ increases or decreases.

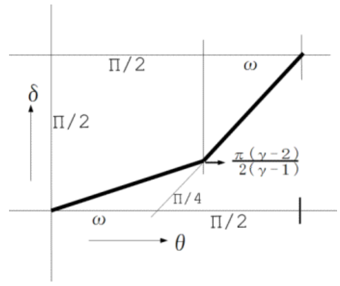

Fig.6

5.3 In the case of $1 < \gamma < 2$, $R = \gamma Rs$, $-\delta = \delta' = \omega - \theta > 0$ in Fig.7, the following formulas are formed.

$(dR'/d\theta)(C'\cos\delta'/R') = -C'\sin\delta$

$d(\log R')/d\theta = -\sin\delta'/\cos\delta' = d(\log(\cos\delta'))/d\delta'$

$\log R' = \log(\cos\delta') + \log A$

$\log A$: Integration constant

$R' = A\cos\delta'$, $\theta = \omega$, $R' = R = A$

$R' = R\cos(\omega - \theta) = \gamma Rs(\cos\delta')$

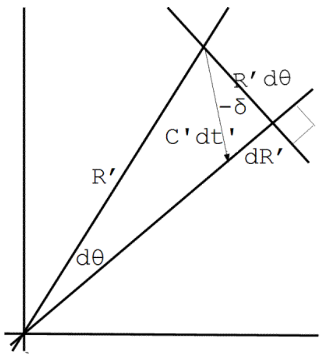

Fig. 7

Then, the formula (6) is rewritten as the following formulas.

(8) $d\omega/d\theta = (2GM/\cos(\omega - \theta)C^2R)/(1 - (2GM/\cos(\omega - \theta)C^2R))$

(9) $d\delta'/d\theta = (\gamma\cos\delta' - 2)/(\gamma\cos\delta' - 1) > 0$

This seems to be a spiral like orbit.

5.4 In the case of $\gamma = 1$ and $R = Rs = 2GM/C2$, the following formulas are formed.

$C'^2 = C^2 - 2GM/Rs = C'^2 - C^2 = 0$

Therefore, the light could not exist in the case of $\gamma < 1$, because C'^2 could not be minus.

6. Conclusions

In the new method, it is assumed that light does not advance straight in gravitational fields, but light advances changing the speed C' according to $C'^2 = C^2 - 2GM/R$ and bending the direction so as to correspond to the acceleration $dU^2/dR \quad (U^2 = 2GM/R)$. Because light's bending angle derived based on the assumptions corresponds to non-linear distortion of radial axis in Schwarzschild's solution, the new method is the same as Schwarzschild's solution. But the new method is simple, easy and exact to calculate light's bending angle comparing to Schwarzschild's solution. And also, light's orbit could be drawn easily and exactly using light's bending angle.

References

1. S. V. Iyer, A. O. Petters. 2007. Gen. Relativ. Gravit. 39, 1563-1582